Math 3E HW #5

Answers must be submitted on Moodle by 11AM on Thursday March 23rd. Do NOT begin the Moodle quiz until you have completed all problems you wish to submit for credit. Moodle will only give you one opportunity to submit your answers. It will also time you out if you leave the quiz open for too long.

Good luck!

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) The columns of $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. 1) _____

Suppose that T is a linear transformation from $\mathcal{R}^3$ into $\mathcal{R}^2$ such that

$T(\mathbf{e}_1) = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$, $T(\mathbf{e}_2) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, and $T(\mathbf{e}_3) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

Find a formula for the image of an arbitrary $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ in $\mathcal{R}^3$.

A) $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6x_1 - 3x_2 \\ 2x_1 \end{bmatrix}$

B) $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6x_1 + 2x_2 - 2x_3 \\ -3x_1 + x_3 \end{bmatrix}$

C) $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6x_1 - 3x_2 \\ 2x_1 \\ 2x_2 + x_3 \end{bmatrix}$

D) $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6x_1 + 2x_2 - 2x_3 \\ 2x_1 \\ -3x_1 + x_3 \end{bmatrix}$

Find the standard matrix of the linear transformation T.

2) T: $\mathcal{R}^2 \to \mathcal{R}^2$ rotates points (about the origin) through $\frac{7}{4}\pi$ radians (with counterclockwise rotation for a positive angle). 2) _____

A) $\begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

B) $\begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{bmatrix}$

C) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

D) $\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

3) T: $\mathcal{R}^2 \to \mathcal{R}^2$ first performs a vertical shear that maps $\mathbf{e}_1$ into $\mathbf{e}_1 + 5\mathbf{e}_2$, but leaves the vector $\mathbf{e}_2$ unchanged, then reflects the result through the horizontal x_1–axis. 3) _____

A) $\begin{bmatrix} -1 & -5 \\ 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 \\ -5 & -1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 5 \\ 0 & -1 \end{bmatrix}$ D) $\begin{bmatrix} -1 & 0 \\ 5 & -1 \end{bmatrix}$

Determine whether the linear transformation T is one–to–one and whether it maps as specified.

4) Let T be the linear transformation whose standard matrix is 4) _____

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & -4 \\ 2 & -2 & -9 \end{bmatrix}.$$

Determine whether the linear transformation T is one–to–one and whether it maps $\mathcal{R}^3$ onto $\mathcal{R}^3$.

A) One–to–one; not onto $\mathcal{R}^3$ B) Not one–to–one; not onto $\mathcal{R}^3$

C) One–to–one; onto $\mathcal{R}^3$ D) Not one–to–one; onto $\mathcal{R}^3$

5) $T(x_1, x_2, x_3) = (-2x_2 - 2x_3, -2x_1 + 8x_2 + 4x_3, -x_1 - 2x_3, 4x_2 + 4x_3)$ 5) _____

Determine whether the linear transformation T is one–to–one and whether it maps $\mathcal{R}^3$ onto $\mathcal{R}^4$.

A) Not one–to–one; onto $\mathcal{R}^4$ B) Not one–to–one; not onto $\mathcal{R}^4$

C) One–to–one; not onto $\mathcal{R}^4$ D) One–to–one; onto $\mathcal{R}^4$

Find the matrix product AB, if it is defined.

6) $A = \begin{bmatrix} -1 & 3 \\ 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 \\ -1 & 4 \end{bmatrix}$. 6) _____

A) $\begin{bmatrix} 2 & 0 \\ -2 & 8 \end{bmatrix}$ B) $\begin{bmatrix} 12 & -1 \\ 8 & -6 \end{bmatrix}$ C) $\begin{bmatrix} -1 & 12 \\ -6 & 8 \end{bmatrix}$ D) $\begin{bmatrix} 2 & -6 \\ -1 & 5 \end{bmatrix}$

7) $A = \begin{bmatrix} 0 & -3 \\ 3 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}$. 7) _____

A) $\begin{bmatrix} -3 & 3 \\ -3 & -9 \end{bmatrix}$ B) $\begin{bmatrix} -6 & -6 \\ 3 & 6 \end{bmatrix}$ C) $\begin{bmatrix} 0 & 6 \\ -3 & 3 \end{bmatrix}$ D) $\begin{bmatrix} 3 & -3 \\ -9 & 3 \end{bmatrix}$

8) $A = \begin{bmatrix} 3 & -2 \\ 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -2 \\ 3 & 6 \end{bmatrix}$. 8) _____

A) $\begin{bmatrix} -18 & -6 \\ -6 & 0 \end{bmatrix}$ B) $\begin{bmatrix} -6 & -18 \\ 0 & -6 \end{bmatrix}$ C) $\begin{bmatrix} -6 & 0 \\ 27 & -6 \end{bmatrix}$ D) $\begin{bmatrix} 0 & 4 \\ 9 & 0 \end{bmatrix}$

9) $A = \begin{bmatrix} -1 & 3 \\ 1 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -2 & 7 \\ 1 & -3 & 2 \end{bmatrix}$. 9) _____

A) $\begin{bmatrix} 0 & -6 \\ 21 & 1 \\ -18 & 12 \end{bmatrix}$

B) $\begin{bmatrix} 3 & 6 & -7 \\ -20 & -1 & 19 \end{bmatrix}$

C) AB is undefined.

D) $\begin{bmatrix} 3 & -7 & -1 \\ 6 & -20 & 19 \end{bmatrix}$

10) $A = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 4 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ -2 & 3 \end{bmatrix}$. 10) _____

A) $\begin{bmatrix} 12 & -6 \\ -8 & 16 \\ 4 & -8 \end{bmatrix}$

B) $\begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix}$

C) AB is undefined.

D) $\begin{bmatrix} 12 & -8 & 4 \\ -6 & 16 & -8 \end{bmatrix}$

11) $A = \begin{bmatrix} 0 & -2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 3 & 2 \\ 0 & -3 & 1 \end{bmatrix}$. 11) _____

A) $\begin{bmatrix} 0 & -2 & 6 \\ -3 & -2 & 7 \end{bmatrix}$

B) AB is undefined.

C) $\begin{bmatrix} 0 & -6 & -12 \\ 0 & -9 & 3 \end{bmatrix}$

D) $\begin{bmatrix} 0 & 6 & -2 \\ -2 & -3 & 7 \end{bmatrix}$

12) $A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 \\ -2 & 1 \\ 0 & 3 \end{bmatrix}$. 12) _____

A) AB is undefined.

B) $\begin{bmatrix} 3 & -6 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

C) $\begin{bmatrix} -3 & -3 \\ 6 & 9 \end{bmatrix}$

D) $\begin{bmatrix} -3 & -3 \\ 9 & 6 \end{bmatrix}$

13) $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & -2 \\ 2 & -2 & 2 \end{bmatrix}$. 13) _____

A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -4 & 4 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 2 & -2 \\ 4 & -4 & 4 \end{bmatrix}$

C) $\begin{bmatrix} 4 & -4 & 4 \\ 1 & 2 & -2 \end{bmatrix}$

D) AB is undefined.