Math 3A HW #7

Answers must be submitted via Moodle before 10AM on Wednesday, April 5th, 2017.

Good luck!

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the absolute extreme values of the function on the interval.

1) \( g(x) = -x^2 + 9x - 18, \ 3 \leq x \leq 6 \)
   
   A) absolute maximum is \( \frac{13}{4} \) at \( x = \frac{11}{2} \); absolute minimum is 0 at \( x = 3 \)
   
   B) absolute maximum is \( \frac{9}{4} \) at \( x = \frac{11}{2} \); absolute minimum is 0 at \( x = 3 \)
   
   C) absolute maximum is \( \frac{9}{4} \) at \( x = \frac{9}{2} \); absolute minimum is 0 at \( x = 3 \)
   
   D) absolute maximum is \( \frac{153}{4} \) at \( x = \frac{9}{2} \); absolute minimum is 0 at \( x = 3 \)

2) \( h(x) = \csc x, \ -\frac{\pi}{2} \leq x \leq \frac{3\pi}{2} \)
   
   A) absolute maximum is \( 1 \) at \( x = \pi \); absolute minimum is \( -1 \) at \( x = \pi \)
   
   B) absolute maximum is \( -1 \) at \( x = \pi \); absolute minimum is \( 1 \) at \( x = 0 \)
   
   C) absolute maximum does not exist; absolute minimum does not exist
   
   D) absolute maximum is \( 0 \) at \( x = -\pi \); absolute minimum is \( -1 \) at \( x = \pi \)

3) \( F(x) = \sqrt[3]{x}, \ -3 \leq x \leq 8 \)
   
   A) absolute maximum is \( 2 \) at \( x = -8 \); absolute minimum is \( 0 \) at \( x = 0 \)
   
   B) absolute maximum is \( 0 \) at \( x = 0 \); absolute minimum is \( 2 \) at \( x = 8 \)
   
   C) absolute maximum is \( 2 \) at \( x = 8 \); absolute minimum is \( -2 \) at \( x = -8 \)
   
   D) absolute maximum is \( 2 \) at \( x = 8 \); absolute minimum is \( 0 \) at \( x = 0 \)

Find the absolute extreme values of the function on the interval.

4) \( f(x) = \tan x, \ -\frac{\pi}{6} \leq x \leq \frac{\pi}{6} \)
   
   A) absolute maximum is \( \sqrt{3} \) at \( x = \frac{\pi}{6} \); absolute minimum is \( \sqrt{3} \) at \( x = -\frac{\pi}{6} \)
   
   B) absolute maximum is \( \sqrt{3} \) at \( x = \frac{2\pi}{6} \); absolute minimum is \( \sqrt{3} \) at \( x = -\frac{\pi}{12} \)
   
   C) absolute maximum is \( \sqrt{3} \) at \( x = \frac{\pi}{6} \); absolute minimum is \( \sqrt{3} \) at \( x = -\frac{\pi}{6} \)
   
   D) absolute maximum is \( \sqrt{3} \) at \( x = \frac{\pi}{6} \) and \( -\frac{\pi}{6} \); no minimum value
5) \( f(x) = |x - 7|, \ 4 \leq x \leq 11 \)
   A) absolute maximum is 4 at \( x = 11 \); absolute minimum is 0 at \( x = 7 \)
   B) absolute maximum is -3 at \( x = 4 \); absolute minimum is 4 at \( x = 11 \)
   C) absolute maximum is 4 at \( x = 11 \); absolute minimum is 3 at \( x = 4 \)
   D) absolute maximum is 3 at \( x = 4 \); absolute minimum is 0 at \( x = 7 \)

6) \( f(x) = x^{4/3}, \ -1 \leq x \leq 8 \)
   A) absolute maximum is 16 at \( x = 8 \); absolute minimum does not exist
   B) absolute maximum is 64 at \( x = 8 \); absolute minimum is 0 at \( x = 0 \)
   C) absolute maximum is 16 at \( x = 8 \); absolute minimum is 0 at \( x = 0 \)
   D) absolute maximum is 16 at \( x = 8 \); absolute minimum is 1 at \( x = -1 \)

7) \( f(x) = 2x^{4/3}, \ -27 \leq x \leq 8 \)
   A) absolute maximum is 162 at \( x = -27 \); absolute minimum is 32 at \( x = 8 \)
   B) absolute maximum is 32 at \( x = 8 \); absolute minimum is 0 at \( x = 0 \)
   C) absolute maximum is 81 at \( x = -27 \); absolute minimum is 0 at \( x = 0 \)
   D) absolute maximum is 162 at \( x = -27 \); absolute minimum is 0 at \( x = 0 \)

8) \( f(x) = \ln(x + 2) + \frac{1}{x}, \ 1 \leq x \leq 5 \)
   A) absolute minimum value is -1 at \( x = -1 \); absolute maximum value is \( \ln 7 + \frac{1}{5} \) at \( x = 5 \)
   B) absolute minimum value is \( \ln 3 + 1 \) at \( x = 1 \); absolute maximum value is \( \ln 7 + \frac{1}{5} \) at \( x = 5 \)
   C) absolute minimum value is \( \ln 4 + \frac{1}{2} \) at \( x = 2 \); absolute maximum value is \( \ln 3 + 1 \) at \( x = 1 \)
   D) absolute minimum value is \( \ln 4 + \frac{1}{2} \) at \( x = 2 \); absolute maximum value is \( \ln 7 + \frac{1}{5} \) at \( x = 5 \)

9) \( f(x) = -5 e^{-x^2}, \ -\infty < x < \infty \)
   A) absolute minimum value is -5 at \( x = 0 \); absolute maximum value is \( -\frac{5}{e} \) at \( x = 1 \)
   B) no minimum value and no maximum value
   C) absolute maximum value is -5 at \( x = 0 \); no minimum value
   D) absolute minimum value is -5 at \( x = 0 \); no maximum value

10) \( f(x) = \ln(-x), \ -3 \leq x \leq -1 \)
    A) absolute minimum value is 0 at \( x = -1 \); no maximum value
    B) absolute minimum value is 0 at \( x = -1 \); absolute maximum value is \( \ln 3 \) at \( x = -3 \)
    C) absolute maximum value is 0 at \( x = -1 \); absolute minimum value is \( -\ln 3 \) at \( x = -3 \)
    D) no minimum value; no maximum value
11) \( f(x) = e^x - x, -1 \leq x \leq 2 \)
   
   A) absolute minimum value is \( e^{-1} + 1 \) at \( x = -1 \); absolute maximum value is \( e^2 - 2 \) at \( x = 2 \)
   
   B) absolute minimum value is 1 at \( x = 0 \); no maximum value
   
   C) absolute minimum value is 1 at \( x = 0 \); absolute maximum value is \( e^2 - 2 \) at \( x = 2 \)
   
   D) absolute minimum value is 1 at \( x = 0 \); absolute maximum value is \( e^{-1} + 1 \) at \( x = -1 \)

Find the largest open interval where the function is changing as requested.

12) Increasing: \( f(x) = \frac{1}{x^2 + 1} \)
   
   A) \((0, \infty)\) \hspace{1cm} B) \((-\infty, 0)\) \hspace{1cm} C) \((-\infty, 1)\) \hspace{1cm} D) \((1, \infty)\)

13) Decreasing: \( f(x) = \frac{x + 6}{x - 3} \)
   
   A) \((-\infty, -3)\) \hspace{1cm} B) \((-\infty, 3) \cup (3, \infty)\) \hspace{1cm} C) \((-\infty, 6) \cup (6, \infty)\) \hspace{1cm} D) none

14) Decreasing: \( y = x^{3/5} + x^{8/5} \)
   
   A) \(\left[0, \frac{3}{8}\right]\) \hspace{1cm} B) \((-\infty, -\frac{3}{8})\) \hspace{1cm} C) \((-\infty, 0) \cup \left(\frac{3}{8}, \infty\right)\) \hspace{1cm} D) \((-\infty, -\frac{3}{8}) \cup (0, \infty)\)

Locate the critical points of the function. Then use the Second Derivative Test to determine (if possible) whether they correspond to local maxima or local minima.

15) \( f(x) = \frac{6x}{x^2 + 1} \)
   
   A) local minimum: \((-1, -3)\), local maximum: \((1, 3)\)
   
   B) local minimum: \((0, 0)\)
   
   C) local minimum: \((1, 0)\), local maximum: \((-1, 0)\)
   
   D) local maximum: \((0, 0)\)

16) \( f(x) = x^3 - 3x^2 + 7 \)
   
   A) local maximum: \((-1, 11)\); local minimum \((1, 5)\)
   
   B) local maximum: \((0, 7)\); local minimum \((2, 3)\)
   
   C) local maximum: \((0, 7)\)
   
   D) local minimum: \((0, 7)\); local maximum: \((2, 5)\)

Use the Concavity Theorem to determine where the given function is concave up and where it is concave down. Also find all inflection points.

17) \( q(x) = 3x^3 + 2x + 8 \)
   
   A) Concave down for all \( x \); no inflection points
   
   B) Concave up on \((0, \infty)\), concave down on \((-\infty, 0)\); inflection point \((0, 8)\)
   
   C) Concave up on \((-\infty, 0)\), concave down on \((0, \infty)\); inflection point \((0, 8)\)
   
   D) Concave up for all \( x \); no inflection points
18) \( f(x) = x^2 - 16x + 69 \)
   A) Concave down for all \( x \); no inflection points
   B) Concave up on \((8, \infty)\), concave down on \((\infty, 8)\); inflection point \((8, 5)\)
   C) Concave up on \((\infty, 8)\), concave down on \((8, \infty)\); inflection point \((8, 5)\)
   D) Concave up for all \( x \); no inflection points

19) \( G(x) = \frac{1}{4}x^4 - x^3 + 14 \)
   A) Concave up on \((\infty, 0) \cup (2, \infty)\), concave down on \((0, 2)\); inflection points \((0, 14)\) and \((2, 10)\)
   B) Concave up for \((\infty, 0)\), concave down for \((0, \infty)\); inflection point \((0, 14)\)
   C) Concave up on \((0, 2)\), concave down on \((\infty, 0) \cup (2, \infty)\); inflection points \((0, 14)\) and \((2, 10)\)
   D) Concave up for \((2, \infty)\), concave down on \((\infty, 2)\); inflection point \((2, 10)\)