Math 3F Fall 2015 Extra Credit Problem Set

P.R. Zulkowski *

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1 Instructions

You do NOT need to attempt all problems to receive partial extra credit. Full credit for attempting any given problem in this set will be given ONLY when your argument is written out neatly, legibly, logically and correctly. I reserve the right to award credit based on my assessment of how well you adhere to these criteria. I reserve the right to alter the content of the problem set below when necessary and at any time prior to the due date of **Wednesday, December 9th**. You may work on these problems in groups, but the work you submit must be your own.

*pzulkowski@peralta.edu

The time-independent Schrodinger equation is given by

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(\boldsymbol{x})\psi = E\psi$$

Here, $V(\boldsymbol{x})$ represents the three-dimensional potential experienced by the particle of mass m. Inside the box, the potential is 0; outside, we assume the potential is $+\infty$. The constant $\hbar$ is called Planck's constant, ψ is called the wavefunction and the constant E is the energy. We can solve this equation explicitly using the technique known as separation of variables.

1. **(3 points)** Assume that $\psi(x, y, z) = X(x)Y(y)Z(z)$; i.e. ψ can be written as the product of three functions each just depending on one spatial coordinate. Using this guess, find three decoupled ordinary differential equations for X, Y and Z.

2. **(6 points)** Assume the box is specified by the region $[0, L_x] \times [0, L_y] \times [0, L_z]$ in $\mathbb{R}^3$. Use the boundary conditions $X(0) = X(L_x) = 0$, $Y(0) = Y(L_y) = 0$, $Z(0) = Z(L_z) = 0$ to obtain an expression for the allowed energy levels $E_{n_x n_y n_z}$. Find $\psi_{n_x n_y n_z}(x, y, z)$ (up to some constant multiple) by explicitly solving for X, Y, and Z.

3. **(3 points)** In order for the function $\psi_{n_x n_y n_z}$ to truly represent a wavefunction, it must be the case that $\int_0^{L_z}\int_0^{L_y}\int_0^{L_x} |\psi_{n_x n_y n_z}|^2 dxdydz = 1$. Then $|\psi_{n_x n_y n_z}|^2$ can be interpreted as a probability density. Find an appropriate scale factor for $\psi_{n_x n_y n_z}$ which satisfies this condition. Finding such a scale factor is called normalizing the wavefunction.

4. **(3 points)** Compute the probability that the particle in the excited state $n_x = 1, n_y = 1, n_z = 2$ can be found in the region $[0, L_x/2] \times [0, L_y/2] \times [0, L_z/2]$