Math 3E Fall 2015 Extra Credit Problem Set

P.R. Zulkowski *

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1 Instructions

You do NOT need to attempt all problems to receive partial extra credit. Full credit for attempting any given problem in this set will be given ONLY when your argument is written out neatly, legibly, logically and correctly. I reserve the right to award credit based on my assessment of how well you adhere to these criteria. I reserve the right to alter the content of the problem set below when necessary and at any time prior to the due date of **Thursday, December 10th**. You may work on these problems in groups, but the work you submit must be your own.

*pzulkowski@peralta.edu

1. **(3 points)** Let V be an n-dimensional (real) vector space and let T be a linear transformation from V into V such that the range and null space of T are identical. Prove that n, the dimension of V is an even integer.

2. **(4 points)** Prove that the eigenvalues of a Hermitian matrix H must be real numbers.

3. **(4 points)** Let V be the real inner product space consisting of the space of real-valued continuous functions on the interval, $-1 \leq t \leq 1$, with the inner product

$$(f, g) = \int_{-1}^{1} f(t)g(t)dt. \tag{1.1}$$

   Let W be the subspace of odd functions, i.e., functions satisfying $f(-t) = -f(t)$. Find the orthogonal complement of W.

4. **(4 points)** Let A be an $n \times n$ matrix and let $p(x)$ be the corresponding characteristic polynomial. Prove that $p(A) = 0$. You may assume that A is diagonalizable.

5. **(4 points)** Compute the Fourier coefficients for $f(t) = \pi - |t - \pi|$ where $f(t) \in C([0, 2\pi])$. Plot the first few Fourier terms. How well do they approximate $f(t)$?