Math 3B Fall 2015 Extra Credit Problem Set

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1 Instructions

You do NOT need to attempt all problems to receive partial extra credit. Full credit for attempting any given problem in this set will be given ONLY when your argument is written out neatly, legibly, logically and correctly. I reserve the right to award credit based on my assessment of how well you adhere to these criteria. I reserve the right to alter the content of the problem set below when necessary and at any time prior to the due date of **Thursday, December 10th**. You may work on these problems in groups, but the work you submit must be your own.

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1. **(3 points)** Find the interval of convergence of $\sum_{n=1}^{\infty} n^3 x^n$ and find its sum.

2. **(3 points)** Show that the Maclaurin series of the function $f(x) = \frac{x}{1-x-x^2}$ is $\sum_{n=1}^{\infty} f_n x^n$ where f_n is the n-th Fibonacci number. By writing $f(x)$ as a sum of partial fractions, find an explicit formula for the n-th Fibonacci number.

3. **(3 points)** Find all functions f that satisfy the equation

$$\left(\int f(x)dx\right)\left(\int \frac{1}{f(x)}dx\right) = -1$$

4. **(3 points)** A uniform disk of radius 1 m is to be cut by a line so that the center of mass of the smaller piece lies halfway along a radius. How close to the center of the disk should the cut be made?

5. **(3 points)** Show that

$$\int_0^1 (1-x^2)^n dx = \frac{2^{2n}(n!)^2}{(2n+1)!}$$