**Math 3C (21945) HW #10**

Due at the beginning of lecture on Thursday, May 7th.

In order to receive a ✓, you must attempt all problems and write out all steps leading to your answers neatly and legibly. You cannot simply write the correct answer to demonstrate your mathematical understanding.

You must include your name, the course title and section number on the first page. All homework sets must be stapled. No late homework will be accepted without my express permission. You may receive a ✗ if these guidelines are not followed.

Good luck!

**Using Green's Theorem, compute the counterclockwise circulation of F around the closed curve C.**

1) $\mathbf{F} = \ln(x^2 + y^2)\mathbf{i} + \tan^{-1}\left(\frac{x}{y}\right)\mathbf{j}$; C is the region defined by the polar coordinate inequalities $8 \le r \le 10$ and $0 \le \theta \le \pi$ 1) ______________

2) $\mathbf{F} = (-4x + 5y)\mathbf{i} + (5x - 6y)\mathbf{j}$; C is the region bounded above by $y = -2x^2 + 80$ and below by $y = 3x^2$ in the first quadrant 2) ______________

**Using Green's Theorem, find the outward flux of F across the closed curve C.**

3) $\mathbf{F} = (x^2 + y^2)\mathbf{i} + (x - y)\mathbf{j}$ ; C is the rectangle with vertices at (0, 0), (9, 0), (9, 6), and (0, 9) 3) ______________

4) $\mathbf{F} = (x - y)\mathbf{i} + (x + y)\mathbf{j}$; C is the triangle with vertices at (0, 0), (7, 0), and (0, 4) 4) ______________

5) $\mathbf{F} = \ln(x^2 + y^2)\,\mathbf{i} + \tan^{-1}\left(\frac{x}{y}\right)\mathbf{j}$; C is the region defined by the polar coordinate inequalities $9 \le r \le 10$ and $0 \le \theta \le \pi$ 5) ______________

**Find the divergence of the field F.**

6) $\mathbf{F} = -14xz^6\mathbf{i} + 5y\mathbf{j} + 2z^7\mathbf{k}$ 6) ______________

7) $\mathbf{F} = \dfrac{y\mathbf{j} - z\mathbf{k}}{(y^2 + z^2)^{1/2}}$ 7) ______________

**Find curl F.**

8) $\mathbf{F}(x, y, z) = 2xyz\mathbf{i} + 8yz\mathbf{j} + 5z\mathbf{k}$ 8) ______________

9) $\mathbf{F}(x, y, z) = e\mathbf{i} + \ln(2y + z)\mathbf{j} + xye^z\mathbf{k}$ 9) ______________

**Parametrize the surface S.**

10) S is the portion of the plane $2x - 2y + 5z = 4$ that lies within the cylinder $x^2 + y^2 = 1$. 10) ______________

11) S is the lower portion of the sphere $x^2 + y^2 + z^2 = 64$ cut by the cone $z = \sqrt{x^2 + y^2}$. 11) ______________

**Calculate the area of the surface S.**

12) S is the portion of the plane $8x + 6y + 4z = 2$ that lies within the cylinder $x^2 + y^2 = 1$. 12) ______________

13) S is the portion of the cylinder $x^2 + y^2 = 16$ that lies between $z = 2$ and $z = 3$. 13) ______________

**Evaluate the surface integral of G over the surface S.**

14) S is the cylinder $y^2 + z^2 = 64$, $z \geq 0$ and $2 \leq x \leq 3$; $G(x, y, z) = z$ 14) ______________

15) S is the hemisphere $x^2 + y^2 + z^2 = 5$, $z \geq 0$; $G(x,y,z) = z^2$ 15) ______________

16) S is the dome $z = 8 - 3x^2 - 3y^2$, $z \geq 0$; $G(x, y, z) = \dfrac{1}{\sqrt{36(x^2 + y^2) + 1}}$ 16) ______________

**Evaluate the surface integral of the function G over the surface S.**

17) $G(x,y,z) = x^2 + y^2 + z^2$; S is the surface of the cube formed from the coordinate planes and the planes $x = 1$, $y = 1$, and $z = 1$ 17) ______________

**Solve the problem.**

18) The shape and density of a thin shell are indicated below. Find the coordinates of the center of mass. 18) ______________

Shell: cylinder $x^2 + z^2 = 16$ bounded by $y = 0$ and $y = 6$
Density: constant

Answer Key
Testname: M3C_HW_10

1) -4
2) 0
3) 432
4) 28
5) 0
6) 5
7) $\dfrac{z^2 - y^2}{(y^2 + z^2)^{3/2}}$
8) $-8y\mathbf{i} + 2xy\mathbf{j} + -2xz\mathbf{k}$
9) $\left(xe^z - \dfrac{1}{2y + z}\right)\mathbf{i} - ye^z\mathbf{j} + 0\mathbf{k}$
10) Answers will vary. One possibility is $\mathbf{r} = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j} + \left(\dfrac{4 - 2r\cos\theta + 2r\sin\theta}{5}\right)\mathbf{k}$, $0 \le r \le 1$, $0 \le \theta \le 2\pi$
11) Answers will vary. One possibility is $\mathbf{r} = 8\cos\phi\,\sin\theta\mathbf{i} + 8\sin\phi\,\sin\theta\mathbf{j} + 8\cos\theta\mathbf{k}$, $0 \le \phi \le 2\pi$, $\dfrac{\pi}{4} \le \theta \le \pi$
12) $\dfrac{1}{2}\sqrt{29}\pi$
13) $8\pi$
14) 128
15) $\dfrac{50}{3}\pi$
16) $\dfrac{8}{3}\pi$
17) 7
18) $\left(0, 3, \dfrac{8}{\pi}\right)$