Math 3C (21945) HW #8

Due at the beginning of lecture on Thursday, April 23rd.

In order to receive a ✓, you must attempt all problems and write out all steps leading to your answers neatly and legibly. You cannot simply write the correct answer to demonstrate your mathematical understanding.

You must include your name, the course title and section number on the first page. All homework sets must be stapled. No late homework will be accepted without my express permission. You may receive a ✗ if these guidelines are not followed.

Good luck!

Evaluate the integral.

1) $\int_0^4 \int_0^3 \int_0^8 xyz \, dx \, dy \, dz$ 1) _____

2) $\int_0^3 \int_0^{4(1 - z/3)} \int_0^{7(1 - y/4 - z/3)} dx \, dy \, dz$ 2) _____

3) $\int_{-1}^1 \int_0^1 \int_0^4 (x^2 + y^2 + z^2) \, dx \, dy \, dz$ 3) _____

Find the volume of the indicated region.

4) the region bounded by the cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $x + z = 9$ 4) _____

5) the region common to the interiors of the cylinders $x^2 + y^2 = 36$ and $x^2 + z^2 = 36$ 5) _____

Evaluate the integral by changing the order of integration in an appropriate way.

6) $\int_0^1 \int_0^4 \int_y^4 \frac{x \sin z}{z} \, dz \, dy \, dx$ 6) _____

7) $\int_0^7 \int_y^7 \int_0^{\pi} \frac{\sin z \sin x}{x} \, dz \, dx \, dy$ 7) _____

8) $\int_0^8 \int_1^2 \int_{\sqrt{x/8}}^1 \frac{e^{y^3}}{z} \, dy \, dz \, dx$ 8) _____

Solve the problem.

9) Set up the triple integral for the volume of the sphere $\rho = 3$ in spherical coordinates. 9) _____

10) Set up the triple integral for the volume of the sphere $\rho = 4$ in cylindrical coordinates. 10) _____

11) Let D be the region that is bounded below by the cone $\phi = \frac{\pi}{4}$ and above by the sphere $\rho = 5$. Set up the triple integral for the volume of D in cylindrical coordinates. 11) _____

Evaluate the spherical coordinate integral.

12) $\int_0^{\pi} \int_0^{\pi} \int_0^{5 \sin \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$ 12) _____

13) $\int_0^{8\pi} \int_0^{\pi} \int_0^{(1 - \cos \phi)/2} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$ 13) _____

Use spherical coordinates to find the volume of the indicated region.

14) the region bounded above by the sphere $x^2 + y^2 + z^2 = 64$ and below by the cone $z = \sqrt{x^2 + y^2}$ 14) _____

15) the region inside the solid sphere $\rho \le 3$ that lies between the cones $\phi = \frac{\pi}{3}$ and $\phi = \frac{\pi}{2}$ 15) _____

16) the region that lies inside the sphere $x^2 + y^2 + z^2 = 49$ and outside the cylinder $x^2 + y^2 = 36$ 16) _____

Change the order of integration and evaluate the integral.

17) $\int_0^{\pi} \int_0^{(1 - \cos \phi)/2} \int_0^{10\pi} \rho^2 \sin \phi \, d\theta \, d\rho \, d\phi$ 17) _____

Solve the problem.

18) Find the center of mass of the rectangular solid of density $\rho(x, y, z) = xyz$ defined by $0 \le x \le 9$, $0 \le y \le 10$, $0 \le z \le 3$. 18) _____

19) Find the center of mass of the region of density $\rho(x, y, z) = \frac{1}{4 - x^2 - y^2}$ bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy–plane. 19) _____

Use the given transformation to evaluate the integral.

20) $x = 8u$, $y = 4v$, $z = 7w$; 20) _____

$$\iiint_R x^2 y^2 \, dx \, dy \, dz,$$

where R is the interior of the ellipsoid $\frac{x^2}{64} + \frac{y^2}{16} + \frac{z^2}{49} = 1$

21) $u = 2x + y - z$, $v = -x + y + z$, $w = -x + y + 2z$; 21) _____

$$\iiint_R (2x + y - z)\, dx\, dy\, dz,$$

where R is the parallelepiped bounded by the planes $2x + y - z = 2$, $2x + y - z = 4$, $-x + y + z = 8$, $-x + y + z = 10$, $-x + y + 2z = 7$, $-x + y + 2z = 10$

1) 1152
2) 14
3) 48
4) 81π
5) 1152
6) $\frac{1 - \cos 4}{2}$
7) $2(1 - \cos 7)$
8) $\frac{8}{3} \ln 2 \, (e - 1)$
9) $\int_0^{2\pi} \int_0^{\pi} \int_0^{3} \varrho^2 \sin\phi \, d\varrho \, d\phi \, d\theta$
10) $\int_0^{2\pi} \int_0^{4} \int_{-\sqrt{16 - r^2}}^{\sqrt{16 - r^2}} r \, dz \, dr \, d\theta$
11) $\int_0^{2\pi} \int_0^{5/\sqrt{2}} \int_r^{\sqrt{25 - r^2}} r \, dz \, dr \, d\theta$
12) $\frac{125}{8}\pi^2$
13) $\frac{4}{3}\pi$
14) $\frac{512}{3}\pi(2 - \sqrt{2})$
15) 9π
16) $\frac{4(343 - 13^{3/2})\pi}{3}$
17) $\frac{5}{3}\pi$
18) $\bar{x} = 6, \bar{y} = \frac{20}{3}, \bar{z} = 2$
19) $\bar{x} = 0, \bar{y} = 0, \bar{z} = \frac{1}{2}$
20) $\frac{128}{15}\pi$
21) 12