Math 3E (21961) HW #8

Due at the beginning of lecture on Thursday, April 23rd.

In order to receive a ✓, you must attempt all problems and write out all steps leading to your answers neatly and legibly. You cannot simply write the correct answer to demonstrate your mathematical understanding.

You must include your name, the course title and section number on the first page. All homework sets must be stapled. No late homework will be accepted without my express permission. You may receive a ✗ if these guidelines are not followed.

Good luck!

Find the eigenvalues of A, and find a basis for each eigenspace.

1) $A = \begin{bmatrix} 2 & 4 \\ -4 & 2 \end{bmatrix}$ 1) _____

Diagonalize the matrix A, if possible. That is, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

2) $A = \begin{bmatrix} 1 & 1 & 4 \\ 0 & -4 & 0 \\ -5 & -1 & -8 \end{bmatrix}$ 2) _____

3) $A = \begin{bmatrix} 6 & 0 & 0 \\ 1 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ 3) _____

4) $A = \begin{bmatrix} -8 & 0 & 0 & 0 \\ 0 & -8 & 0 & 0 \\ 1 & -4 & 8 & 0 \\ -1 & 2 & 0 & 8 \end{bmatrix}$ 4) _____

5) $A = \begin{bmatrix} -3 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ -12 & 3 & -9 & 12 \\ 0 & 0 & 0 & -3 \end{bmatrix}$ 5) _____

Solve the initial value problem.

6) $\mathbf{x}' = A\mathbf{x}$, $x(0) = \begin{bmatrix} 3 \\ 3.2 \end{bmatrix}$, where $A = \begin{bmatrix} -4 & -3.125 \\ 8 & -4 \end{bmatrix}$ 6) _____

Answer Key
Testname: M3E_21961_HW_8

1) $2-4i, \left\{\begin{bmatrix} 1 \\ -i \end{bmatrix}\right\}; 2+4i, \left\{\begin{bmatrix} 1 \\ i \end{bmatrix}\right\}$

2)
$$P = \begin{bmatrix} 1 & 0 & -1 \\ -9 & -4 & 0 \\ 1 & 1 & 1 \end{bmatrix}, D = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

3) Not diagonalizable

4)
$$P = \begin{bmatrix} 16 & 32 & 0 & 0 \\ 8 & 8 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} -8 & 0 & 0 & 0 \\ 0 & -8 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$

5)
$$P = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ -2 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} -3 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -9 \end{bmatrix}$$

6) $\mathbf{x}(t) = \begin{bmatrix} -2\sin 5t + 3\cos 5t \\ 3.2\cos 5t + 4.8\sin 5t \end{bmatrix} e^{-4t}$