Math 3E (21961) HW #6

Due at the beginning of lecture on Thursday, March 26th.

In order to receive a ✓, you must attempt all problems and write out all steps leading to your answers neatly and legibly. You cannot simply write the correct answer to demonstrate your mathematical understanding.

You must include your name, the course title and section number on the first page. All homework sets must be stapled. No late homework will be accepted without my express permission. You may receive a ✗ if these guidelines are not followed.

Good luck!

The vector x is in a subspace H with a basis $\beta = \{\mathbf{b}_1, \mathbf{b}_2\}$. Find the β–coordinate vector of x.

1) $\mathbf{b}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 17 \\ -13 \end{bmatrix}$ 1) _____

2) $\mathbf{b}_1 = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 6 \\ 1 \\ -3 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 26 \\ 9 \\ -23 \end{bmatrix}$ 2) _____

Determine the rank of the matrix.

3) $\begin{bmatrix} 1 & -2 & 2 & -4 \\ 2 & -4 & 7 & -4 \\ -3 & 6 & -6 & 12 \end{bmatrix}$ 3) _____

4) Let $H = \left\{ \begin{bmatrix} a + 3b + 4d \\ c + d \\ -3a - 9b + 4c - 8d \\ -c - d \end{bmatrix} : a, b, c, d \text{ in } \mathbb{R} \right\}$ 4) _____

Find the dimension of the subspace H.

5) If A is a 5×9 matrix, what is the smallest possible dimension of Nul A? 5) _____

Determine if the vector u is in the column space of matrix A and whether it is in the null space of A.

6) $\mathbf{u} = \begin{bmatrix} 5 \\ -3 \\ 5 \end{bmatrix}$, $A = \begin{bmatrix} 1 & -3 & 4 \\ -1 & 0 & -5 \\ 3 & -3 & 6 \end{bmatrix}$ 6) _____

Find the dimensions of the null space and the column space of the given matrix.

7) $A = \begin{bmatrix} 1 & -5 & -4 & 3 & 0 \\ -2 & 3 & -1 & -4 & 1 \end{bmatrix}$ 7) _____

8) $A = \begin{bmatrix} 1 & -2 & 3 & 1 & 0 & 5 & -4 \\ 0 & 0 & 1 & -6 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ 8) _____

Compute the determinant of the matrix by cofactor expansion/row reduction.

9) $\begin{bmatrix} 8 & 0 & 0 \\ 7 & -4 & 0 \\ -4 & 2 & 2 \end{bmatrix}$ 9) _____

10) $\begin{bmatrix} -1 & 2 & -1 \\ 4 & 3 & -2 \\ -3 & -4 & -1 \end{bmatrix}$ 10) _____

11) $\begin{bmatrix} -5 & 5 & -5 & 3 \\ 0 & -1 & 2 & -2 \\ 0 & 3 & 0 & 0 \\ 0 & -3 & 1 & 4 \end{bmatrix}$ 11) _____

12) $\begin{bmatrix} 4 & 1 & 2 \\ -4 & -1 & -5 \\ 8 & 5 & 3 \end{bmatrix}$ 12) _____

13) $\begin{bmatrix} 2 & 1 & -3 & 1 \\ 0 & 6 & -1 & 6 \\ -4 & 4 & 5 & 4 \\ -2 & 5 & 1 & 3 \end{bmatrix}$ 13) _____

Answer Key
Testname: M3E_21961_HW_6

1) $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$
2) $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$
3) 2
4) dim H = 2
5) 4
6) In Col A, not in Nul A
7) dim Nul A = 3, dim Col A = 2
8) dim Nul A = 4, dim Col A = 3
9) −64
10) 38
11) 150
12) 36
13) 0