

Math 3C (21945) HW #5

Due at the beginning of lecture on Thursday, March 12th.

In order to receive a ✓, you must attempt all problems and write out all steps leading to your answers neatly and legibly. You cannot simply write the correct answer to demonstrate your mathematical understanding.

You must include your name, the course title and section number on the first page. All homework sets must be stapled. No late homework will be accepted without my express permission. You may receive a ✗ if these guidelines are not followed.

Good luck!

Compute the gradient of the function at the given point.

1) $f(x, y) = \ln(8x - 8y)$, $(4, 8)$ 1) _____

2) $f(x, y, z) = \tan^{-1} \frac{2x}{8y + 7z}$, $(6, 0, 0)$ 2) _____

Find the derivative of the function at the given point in the direction of A.

3) $f(x, y) = \ln(-4x - 5y)$, $(-7, 2)$, $A = 6\mathbf{i} + 8\mathbf{j}$ 3) _____

4) Find the direction in which the function is increasing most rapidly at the point P_0 . 4) _____

$f(x, y, z) = x\sqrt{y^2 + z^2}$, $P_0(2, 1, 1)$

5) Find the derivative of the function $f(x, y) = e^{xy}$ at the point $(0, 7)$ in the direction in which the function increases most rapidly. 5) _____

6) Find the derivative of the function $f(x, y, z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ at the point $(-8, 8, -8)$ in the direction in which the function decreases most rapidly. 6) _____

Find all local extreme values of the given function and identify each as a local maximum, local minimum, or saddle point.

7) $f(x, y) = 2xy - 5x + 5y$ 7) _____

8) $f(x, y) = -5xy(x + y) + 2$ 8) _____

9) $f(x, y) = 9 - x^4y^4$ 9) _____

Find the absolute maxima and minima of the function on the given domain.

10) $f(x, y) = 3x + 6y$ on the closed triangular region with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$ 10) _____

11) $f(x, y) = 3x^2 + 10y^2$ on the closed triangular region bounded by the lines $y = x$, $y = 2x$, and $x + y = 6$ 11) _____

Find the extreme values of the function subject to the given constraint.

12) $f(x, y) = x^2 + y^2, \quad xy^2 = 128$

12) _____

13) $f(x, y) = x^2y, \quad x^2 + 2y^2 = 6$

13) _____

14) $f(x, y, z) = x^3 + y^3 + z^3, \quad x^2 + y^2 + z^2 = 4$

14) _____

Answer Key

Testname: M3C_HW_5

1) $-\frac{1}{4}\mathbf{i} + \frac{1}{4}\mathbf{j}$

2) $-\frac{2}{3}\mathbf{j} - \frac{7}{12}\mathbf{k}$

3) $-\frac{16}{45}$

4) $\sqrt{3}\left(\frac{\mathbf{i}}{3} + \frac{\mathbf{j}}{3} + \frac{\mathbf{k}}{3}\right)$

5) 7

6) $-\frac{1}{4}\sqrt{2}$

7) $f\left(-\frac{5}{2}, \frac{5}{2}\right) = \frac{25}{2}$, saddle point

8) $f(0, 0) = 2$, test is inconclusive

9) $f(0, 0) = 9$, local maximum

10) Absolute maximum: 6 at (0, 1); absolute minimum: 0 at (0, 0)

11) Absolute maximum: 172 at (2, 4); absolute minimum: 0 at (0, 0)

12) Maximum: none; minimum: 48 at $(4, \pm 4\sqrt{2})$

13) Maximum: 4 at $(\pm 2, 1)$; minimum: -4 at $(\pm 2, -1)$

14) Maximum: 8 at (2, 0, 0), (0, 2, 0), (0, 0, 2); minimum: -8 at (-2, 0, 0), (0, -2, 0), (0, 0, -2)