Math 3C (21945) HW #4

Due at the beginning of lecture on Thursday, March 5th.

In order to receive a ✓, you must attempt all problems and write out all steps leading to your answers neatly and legibly. You cannot simply write the correct answer to demonstrate your mathematical understanding.

You must include your name, the course title and section number on the first page. All homework sets must be stapled. No late homework will be accepted without my express permission. You may receive a ✗ if these guidelines are not followed.

Good luck!

Find all the first order partial derivatives for the following function.

1) $f(x, y) = (5x^4y^3 + 6)^2$ 1) _____

2) $f(x, y) = \dfrac{1}{\sqrt{x^2 + y^2}}$ 2) _____

3) $f(x, y) = \ln\left(\dfrac{y^7}{x^7}\right)$ 3) _____

4) $f(x, y, z) = x^2y + y^2z + xz^2$ 4) _____

5) $f(x, y, z) = \ln (xy)^z$ 5) _____

Find all the second order partial derivatives of the given function.

6) $f(x, y) = x^2 + y - e^{x+y}$ 6) _____

Determine whether the function is a solution of the wave equation.

7) $w(x, t) = \sin(-10x - 10ct)$ 7) _____

8) $w(x, t) = e^{-4ct} \cos -4x$ 8) _____

Determine whether the given function satisfies a Laplace equation.

9) $f(x, y, z) = 7x^2 - 5y^2 - 2z^2$ 9) _____

10) $f(x, y) = \dfrac{x^2}{y}$ 10) _____

Solve the problem.

11) Evaluate $\dfrac{\partial w}{\partial u}$ at $(u,v) = (5, 3)$ for the function $w(x, y) = xy - y^2$; $x = u - v$, $y = uv$. 11) _____

12) Evaluate $\frac{\partial u}{\partial z}$ at $(x, y, z) = (3, 3, 5)$ for the function $u(p, q, r) = p^2q^2 - r$; $p = y - z$, $q = x + z$, $r = x + y$. 12) _____

13) Evaluate $\frac{\partial u}{\partial y}$ at $(x, y, z) = (2, 3, 0)$ for the function $u(p, q, r) = e^{pq} \cos(r)$; $p = \frac{1}{x}$, $q = x^2 \ln y$, $r = z$. 13) _____

Use implicit differentiation to find the specified derivative at the given point.

14) Find $\frac{\partial z}{\partial y}$ at the point $(7, 1, -1)$ for $\ln\left(\frac{yz}{x}\right) - e^{xy+z^2} = 0$. 14) _____

Solve the problem.

15) Find the equation for the tangent plane to the surface $z = -8x^2 - 10y^2$ at the point $(2, 1, -42)$. 15) _____

16) Find the equation for the tangent plane to the surface $x^2 + 6xyz + y^2 = -4z^2$ at the point $(-1, -1, -1)$. 16) _____

Answer Key
Testname: M3C_HW_4

1) $\frac{\partial f}{\partial x} = 40x^3y^3(5x^4y^3 + 6)$; $\frac{\partial f}{\partial y} = 30x^4y^2(5x^4y^3 + 6)$

2) $\frac{\partial f}{\partial x} = -\left(\frac{x}{(x^2 + y^2)^{3/2}}\right)$; $\frac{\partial f}{\partial y} = -\left(\frac{y}{(x^2 + y^2)^{3/2}}\right)$

3) $\frac{\partial f}{\partial x} = -\frac{7}{x}$; $\frac{\partial f}{\partial y} = \frac{7}{y}$

4) $\frac{\partial f}{\partial x} = 2xy + z^2$; $\frac{\partial f}{\partial y} = x^2 + 2yz$; $\frac{\partial f}{\partial z} = y^2 + 2xz$

5) $\frac{\partial f}{\partial x} = \frac{z}{x}$; $\frac{\partial f}{\partial y} = \frac{z}{y}$; $\frac{\partial f}{\partial z} = \ln(xy)$

6) $\frac{\partial^2 f}{\partial x^2} = 2 - e^{x+y}$; $\frac{\partial^2 f}{\partial y^2} = -e^{x+y}$; $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -e^{x+y}$

7) Yes
8) No
9) Yes
10) No
11) -69
12) 320
13) 6

14) $\frac{1 - 7e^8}{1 - 2e^8}$

15) $-32x - 20y - z = -42$
16) $4x + 4y - 2z = -6$