Math 3E (21961) HW #4

Due at the beginning of lecture on Thursday, March 5th.

In order to receive a ✓, you must attempt all problems and write out all steps leading to your answers neatly and legibly. You cannot simply write the correct answer to demonstrate your mathematical understanding.

You must include your name, the course title and section number on the first page. All homework sets must be stapled. No late homework will be accepted without my express permission. You may receive a ✗ if these guidelines are not followed.

Good luck!

1) Define $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. 1) _____

Suppose that T is a linear transformation from $\mathcal{R}^3$ into $\mathcal{R}^2$ such that

$T(\mathbf{e}_1) = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$, $T(\mathbf{e}_2) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, and $T(\mathbf{e}_3) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

Find a formula for the image of an arbitrary $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ in $\mathcal{R}^3$.

Find the standard matrix of the linear transformation T.

2) T: $\mathcal{R}^2 \to \mathcal{R}^2$ first maps $\mathbf{e}_1$ into $\mathbf{e}_1 + 5\mathbf{e}_2$, but leaves the vector $\mathbf{e}_2$ unchanged, then reflects the result through the horizontal x_1–axis. 2) _____

Determine whether the linear transformation T is one–to–one and whether it maps as specified.

3) Let T be the linear transformation whose standard matrix is 3) _____

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & -4 \\ 2 & -2 & -9 \end{bmatrix}.$$

Determine whether the linear transformation T is one–to–one and whether it maps $\mathcal{R}^3$ onto $\mathcal{R}^3$.

4) $T(x_1, x_2, x_3) = (-2x_2 - 2x_3, -2x_1 + 8x_2 + 4x_3, -x_1 - 2x_3, 4x_2 + 4x_3)$ 4) _____

Determine whether the linear transformation T is one–to–one and whether it maps $\mathcal{R}^3$ onto $\mathcal{R}^4$.

Perform the matrix operation.

5) Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 6 \end{bmatrix}$. Find $4A + B$. 5) _____

Find the matrix product AB, if it is defined.

6) $A = \begin{bmatrix} 3 & -2 \\ 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -2 \\ 3 & 6 \end{bmatrix}$. 6) _____

7) $A = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 4 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ -2 & 3 \end{bmatrix}$. 7) _____

8) $A = \begin{bmatrix} 0 & -2 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 3 & 2 \\ 0 & -3 & 1 \end{bmatrix}$. 8) _____

Find the transpose of the matrix.

9) $\begin{bmatrix} 4 & 6 \\ -6 & 0 \\ -5 & 5 \end{bmatrix}$ 9) _____

1) $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6x_1 + 2x_2 - 2x_3 \\ -3x_1 + x_3 \end{bmatrix}$

2) $\begin{bmatrix} 1 & 0 \\ -5 & -1 \end{bmatrix}$

3) One-to-one; onto $\mathcal{R}^3$

4) Not one-to-one; not onto $\mathcal{R}^4$

5) $\begin{bmatrix} 4 & 16 \\ 7 & 30 \end{bmatrix}$

6) $\begin{bmatrix} -6 & -18 \\ 0 & -6 \end{bmatrix}$

7) AB is undefined.

8) $\begin{bmatrix} 0 & 6 & -2 \\ -2 & -3 & 7 \end{bmatrix}$

9) $\begin{bmatrix} 4 & -6 & -5 \\ 6 & 0 & 5 \end{bmatrix}$