Math 3C (21945) HW #3

Due at the beginning of lecture on Thursday, February 26th.

In order to receive a ✓, you must attempt all problems and write out all steps leading to your answers neatly and legibly. You cannot simply write the correct answer to demonstrate your mathematical understanding.

You must include your name, the course title and section number on the first page. All homework sets must be stapled. No late homework will be accepted without my express permission. You may receive a ✗ if these guidelines are not followed.

Good luck!

Calculate the arc length of the indicated portion of the curve r(t).

1) $\mathbf{r}(t) = 3t\mathbf{i} + \left(8\cos\frac{1}{2}t\right)\mathbf{j} + \left(8\sin\frac{1}{2}t\right)\mathbf{k}$; $-9 \le t \le -3$ 1) _____

2) $\mathbf{r}(t) = \frac{16}{3}\sqrt{2}t^{3/2}\mathbf{i} + (8t\sin t)\mathbf{j} + (8t\cos t)\mathbf{k}$; $-3 \le t \le 7$ 2) _____

Find the curvature κ for the given function.

3) $\mathbf{r}(t) = (3t\sin t + 3\cos t)\mathbf{i} + 3\mathbf{j} + (3t\cos t - 3\sin t)\mathbf{k}$ 3) _____

4) $\mathbf{r}(t) = -3\mathbf{i} + (10 + 2t)\mathbf{j} + (t^2 + 4)\mathbf{k}$ 4) _____

Find T, N, and B for the given space curve.

5) $\mathbf{r}(t) = (t^2 - 2)\mathbf{i} + (2t - 5)\mathbf{j} + 5\mathbf{k}$ 5) _____

Find the point of the curve at which the curvature is at a maximum.

6) $y = x^2 - 6x$ 6) _____

Find the domain and range and describe the level curves for the function f(x,y).

7) $f(x, y) = (7x - 3y)^5$ 7) _____

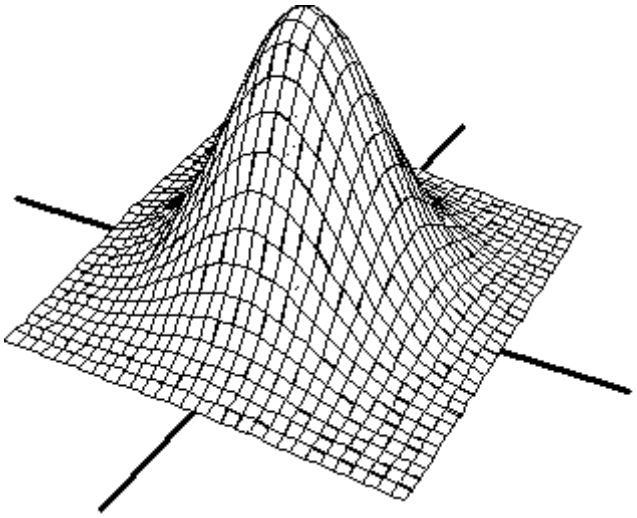
8) $f(x, y) = \frac{y + 1}{x^2}$ 8) _____

9) $f(x, y) = \sin^{-1}(x^2 + y^2)$ 9) _____

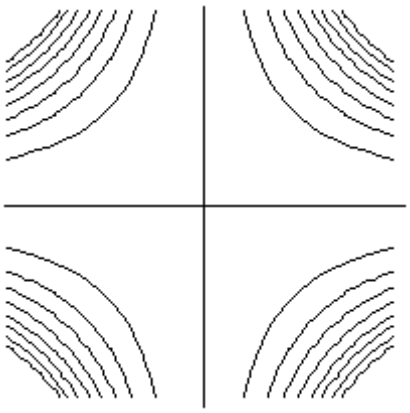
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Match the surface show below to the graph of its level curves.

10) 10) _____



A)



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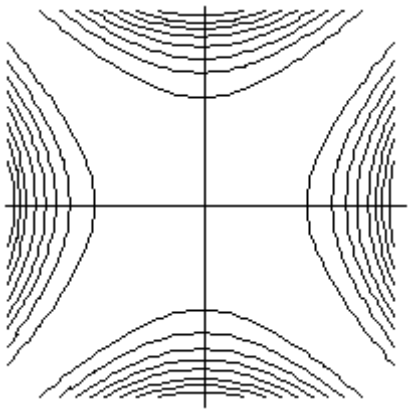
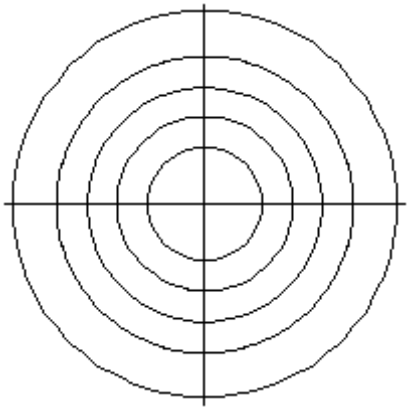
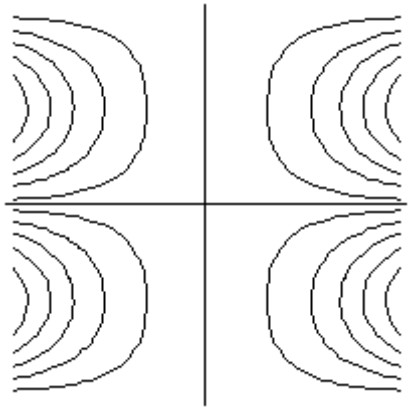


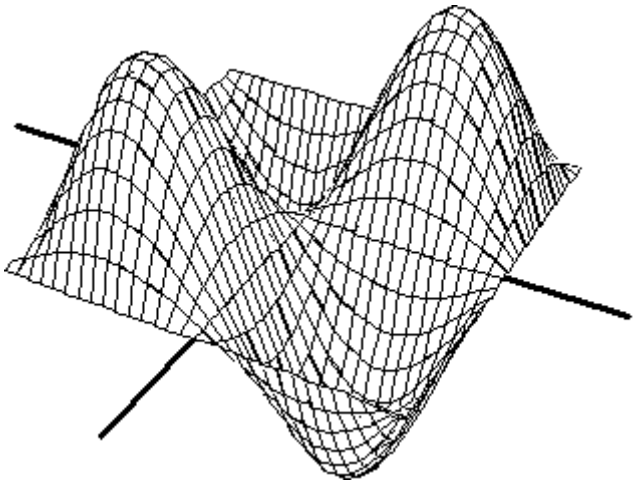
C)



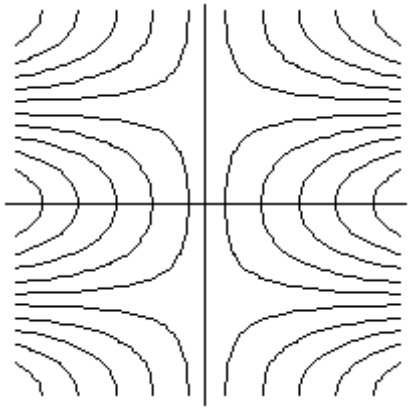
D)



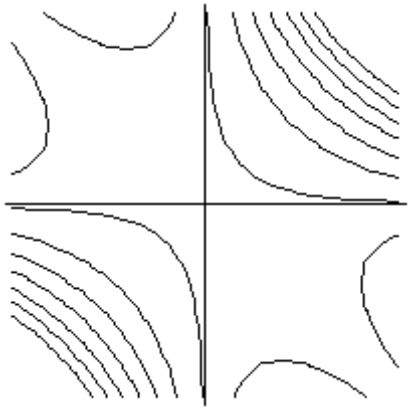
11) 11) _____



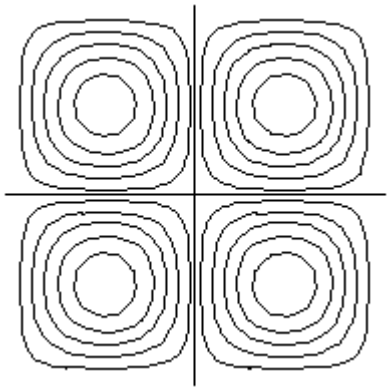
A)



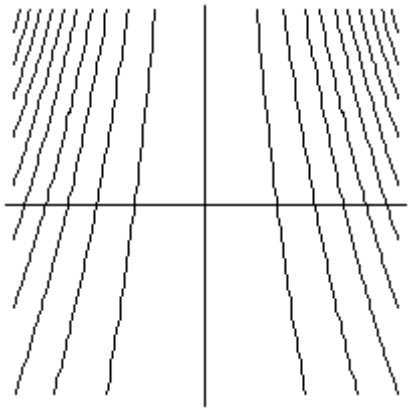
B)



C)



D)



Find two paths of approach from which one can conclude that the function has no limit as (x, y) approaches (0, 0).

12) $f(x, y) = \frac{7y}{\sqrt{6x^2 + 5y^2}}$ 12) _____

13) $f(x, y) = \frac{|xy|}{xy}$ 13) _____

14) $f(x, y) = \dfrac{x^2}{x^4 + y^2}$ 14) _____

Find the limit.

15) $\lim\limits_{(x, y) \to (0, 0)} \dfrac{10x^2 + 8y^2 + 2}{10x^2 - 8y^2 + 10}$ 15) _____

16) $\lim\limits_{(x, y) \to (0, 0)} \sin\left(\dfrac{x^5 + y^9}{x - y + 10}\right)$ 16) _____

Use polar coordinates to find the limit of the function as (x, y) approaches (0, 0).

17) $f(x, y) = \cos^{-1}\left(\dfrac{x^3 - xy^2}{x^2 + y^2}\right)$ 17) _____

1) 30
2) 240
3) $\kappa = \frac{1}{3t}$
4) $\kappa = \frac{1}{2(t^2 + 1)^{3/2}}$
5) $\mathbf{T} = \frac{t}{\sqrt{t^2 + 1}}\mathbf{i} + \frac{1}{\sqrt{t^2 + 1}}\mathbf{j}$; $\mathbf{N} = \frac{1}{\sqrt{t^2 + 1}}\mathbf{i} - \frac{t}{\sqrt{t^2 + 1}}\mathbf{j}$; $\mathbf{B} = -\mathbf{k}$
6) (3, –9)
7) Domain: all points in the xy–plane; range: all real numbers; level curves: lines $7x - 3y = c$
8) Domain: all points in the xy–plane excluding $x = 0$; range: all real numbers; level curves: parabolas $y = cx^2 - 1$
9) Domain: all points in the xy–plane satisfying $x^2 + y^2 \le 1$; range: real numbers $-\frac{\pi}{2} \le z \le \frac{\pi}{2}$; level curves: circles with centers at (0,0) and radii r, $0 < r \le 1$
10) C
11) C
12) Answers will vary. One possibility is Path 1: $x = t$, $y = t$; Path 2: $x = 0$, $y = t$
13) Answers will vary. One possibility is Path 1: $x = t$, $y = t$; Path 2: $x = t$, $y = -t$
14) Answers will vary. One possibility is Path 1: $x = t$, $y = t$; Path 2: $x = t$, $y = 2t$
15) $\frac{1}{5}$
16) 0
17) $\frac{\pi}{2}$