Math 1 Spring 2015 Extra Credit Problem Set

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1 Instructions

You do NOT need to attempt all problems to receive partial extra credit. Full credit for attempting any given problem in this set will be given ONLY when your argument is written out neatly, legibly, logically and correctly. I reserve the right to award credit based on my assessment of how well you adhere to these criteria. I reserve the right to alter the content of the problem set below when necessary and at any time prior to the due date of **Wednesday, May 13th**. You may work on these problems in groups, but the work you submit must be your own.

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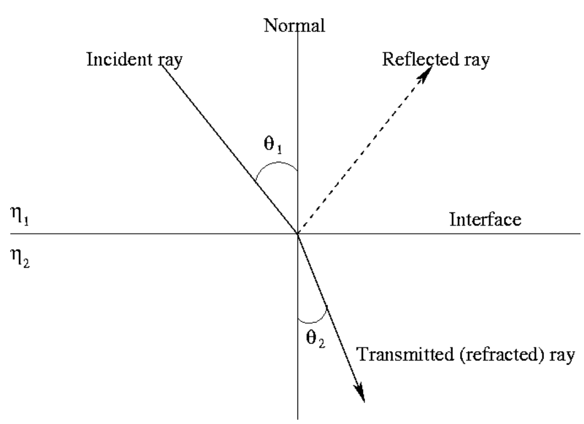
1. **(2 points)** Prove that the following equation is an identity:

$$\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)=\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$$

2. **(2 points)** Snell's law describes how light bends when it passes from one material of index of refraction n_1 into a new material of index of refraction n_2. Let θ_1 be the angle of incidence and let θ_2 be the angle of refraction. Then Snell's law states that $n_1\sin(\theta_1)=n_2\sin(\theta_2)$ (see picture).

   Assume $n_{water}=1.33, n_{air}=1$. At what angle of incidence should a beam of light strike the surface of a still pond if the angle between the reflected ray and the refracted ray is to be 90°?



3. **(3 points)** Compute the instantaneous rate of change $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ for $f(x) = 2x^2 + x + 1$.

4. **(4 points)** Compute the instantaneous rate of change $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ for $f(x) = \sqrt{x}$.

5. **(3 points)** Let A_n be the area of a polygon with n equal sides inscribed in a circle of radius r. By dividing the polygon into n congruent triangles with central angle $\frac{2\pi}{n}$, show that $A_n = \frac{1}{2}nr^2 \sin\left(\frac{2\pi}{n}\right)$. If $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$, show that $\lim_{n\to\infty} A_n = \pi r^2$.

6. **(1 point)** Compute the first few terms of the sequence $a_{n+1} = \frac{1}{2}a_n + \frac{1}{a_n}$ with $a_0 = 1$. What number does this sequence appear to approach?